MASENO SCHOOL

Topic 8: Formulae and Variation

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Formulae

1. Make n the subject of the formula : $m=\sqrt[3]{\dfrac{ax^2-n}{w-n}}$

2. Make M the subject of the formula : $X=\dfrac{-MN}{\sqrt{M^2+N}}$

3. Make x the subject of the formula : $x+r=\sqrt{\dfrac{(2x+r)^2}{4}}$

4. Make R the subject of the formula : $A=\pi\left(R^2-r^2\right)$ where A is the area between two concentric circles of radii R and r.

5. Given that $L^{\frac{1}{2}}=\dfrac{Pt}{P-t}$ and that $P=\dfrac{L^2}{x}$, write t in terms of x and L.

6. Make x the subject of the formula : $\dfrac{a^2}{b}=a\sqrt{\dfrac{x^2-m}{m}}$

7. Express h in terms of E and X only in the formula : $\dfrac{E}{X}=\sqrt{\dfrac{h-0.5}{1-h}}$

8. Make R the subject of the formula : $m=\dfrac{CR}{\sqrt[3]{R^3-C}}$

9. Make b the subject of the formula : $t=\sqrt{\dfrac{a-b^2}{1+ab^2}}$

10. Given that $t=\sqrt[3]{\dfrac{ay-bx}{mx-n}}$, find an expression for the reciprocal of x.

11. Make k the subject of the formula and simplify : $t=\dfrac{1+2y}{\sqrt{k+2ky}}$

12. Make c the subject of the formula $\dfrac{c}{A^2\sqrt{(c-b)(c+b)}}=1$

13. Make x the subject of the formula : $P=\dfrac{1}{2}\sqrt{\dfrac{x+2w}{4x+3r}}$

14. Make x the subject of the formula $L=\dfrac{2}{3}\sqrt{\dfrac{x^2-PT}{y}}$

15. Make Q the subject of the formula $T=P\sqrt{\dfrac{Q^2}{Q^2-1}}$

16. Make p the subject of the formula $d=\sqrt[3]{\dfrac{q}{q-A^p}}$

17. Make P the subject of the formula $\mathrm{T}=\dfrac{-\mathrm{PK}}{\sqrt{\mathrm{P}^2\mathrm{K}+1}}$

18. Make p the subject of the formula $p^2=(p-q)(p-r)$

19. Express p in terms of q and r in the formula : $r^{p+1}=1+3q$

20. Given that $a^{x+1}-a^{x-1}=a-1$, show that $x=1-\log_a(a+1)$

Variation: Section I

1. P varies directly as the square of Q and inversely as the square root of R. If Q is reduced by 12% while R is increased by 21%, find the percentage change in P.

2. The distance *s* metres of an object varies partly with time *t* seconds and partly with the square root of time. Given that $s=16$ when $t=4$ and $s=48$ when $t=16$, write an equation connecting *s* and *t* hence find s when $t=9$.

3. A quantity P varies jointly as the square root of Q and as the inverse of the square of R. Determine the percentage change in P when Q is increased by 44% and R decreased by 28%.

4. The mass of wire m grams (g) is partly constant and partly varies as the square of its thickness t mm. When t = 2 mm, m = 40 g and when t = 4 mm, m = 100 g. Determine the value of m when t = 7 mm.

5. The quantities P, Q and R are such that P varies directly as the square of Q and inversely as the square root of R. P = 8 when Q = 2 and R = 9. Determine the equation connecting P, Q and R.

6. Two variables A and B are such that A varies partly as B and partly as the square root of B. Given that A = 8.58 when B = 1.69 and A = 9.52 when B = 1.96, find the law connecting A and B hence determine A when B = 2.89.

7. A quantity Q is partly constant and partly varies as the square of E. When $E=2$, $Q=560$ and when $E=3$, $Q=510$. Find an equation connecting Q and E hence determine E given $Q=537.5$.

8. A quantity P is partly constant and partly varies as the inverse of Q^2. When $Q=2$, $P=49\frac{1}{4}$ and when $Q=3$, $P=49\frac{2}{3}$. Determine the value of P when $Q=5$.

9. A quantity y varies inversely as the square of x. The difference between the value of y when $x=6$ and when $x=10$ is 16. Find the law connecting x and y.

10. Three quantities P, Q and R are such that P varies as the square of Q and inversely as the square root of R. Given that $P=20$ when $Q=5$ and $R=9$, find P when $Q=7$ and $R=25$.

11. H varies as V and inversely as the square of r. Find the percentage change in H if V is increased by 20% and at the same time r is increased by 50%.

12. Two quantities M and N are such that M varies partly as N and partly as the square of N. Determine the relationship between M and N given that when $M=1050$, $N=10$ and when $M=1272$, $N=12$.

13. Two variables m and n are such that m is directly proportional to x and n is inversely proportional to x. When $x=2$, their sum is 8 and when $x=3$, their sum is 7. Determine the relationship between m and n.

14. The mass of a cylinder m varies jointly as the square of the radius r and as the height h. If the radius is increased by 20% and the height by 10%, find the percentage increase in mass.

15. The cost C of producing N items varies partly as N and partly as the inverse of N. To produce 25 items, it costs Ksh. 1350 and to produce 30 items, it costs Ksh 1400. Find the cost of producing 40 items.

16. A quantity v is partly constant and partly varies as u. If $u=1$ when $v=12$ and $u=3$ when $v=22$, find the value of v when $u=5$.

17. Three quantities x, y and z are such that x varies directly as the square of y and inversely as the square root of z. Given that y is increased by 5% and z decreased by 36%, find the percentage change in z.

18. Three variables P, Q and R are such that P varies as the square of Q and inversely as R. If Q is halved and R is doubled, find the percentage change in P.

19. A quantity A is partly constant and partly varies inversely as a quantity B. Given that $A=-10$ when $B=2.5$ and $A=10$ when $B=1.25$, find the value of B when $A=32.5$.

20. Three variables r, t and s are such that t varies inversely as the cube of r and r varies as the square root of s. Given that $t=2.5$ when $s=4$, find the law connecting t and s.

Variation: Section II

1. Three quantities R, S and T are such that R varies directly as S and inversely as the square root of T.

(a) $R=480$ when $S=150$ and $T=25$, write an equation connecting R, S and T. *(4 marks)*

(b) Find:

(i) The value of R when $S=360$ and $T=2.25$. *(2 marks)*

(ii) The percentage change in R if S is increased by 5% and T decreased by 15.36%. *(4 marks)*

2. The cost C of producing n items varies partly as n and as the inverse of n. To produce two items, it cost Ksh. 270 and to produce three items it costs Ksh. 280. Find:

(a) The law connecting C and n. *(4 marks)*

(b) The cost of producing 10 items. *(3 marks)*

(c) The number of items produced at a cost of Ksh. 920. *(3 marks)*

3. Two variables A and B are such that A partly varies as B and partly as the square root of B. Given that $A=30$ when $B=9$ and $A=70$ when $B=25$;

(a) Find the law connecting A and B. *(4 marks)*

(b) Calculate the value of:

(i) A when $B=36$. *(2 marks)*

(ii) B when $A=17.22$. *(4 marks)*

4. A class is planning a field trip to an art gallery. The cost of renting a bus is Ksh. 25000. There is an additional cost of Ksh. 400 per student for the entrance fee.

(a) Identify the fixed cost and the variable cost for this partial variation. *(2 marks)*

(b) Write an equation relating the cost, C, in Ksh, and the number of students, n. *(1 mark)*

(c) Use your equation in (b) above to find:

(i) The total cost if 25 students attend. *(2 marks)*

(ii) The total number of students to attend with a budget of Ksh. 40000. *(2 marks)*

(d) If the cost of renting the bus is increased by Ksh. 3000 and the entrance fee decreased in the ratio $3:4$, find the number of students that should attend the trip for which the total cost would be unchanged. *(3 marks)*

5. P varies directly as the square of Q and inversely as R.

(a) If Q increases by 20% and R decreases by 10%, find the percentage change in P. *(3 marks)*

(b) Given that $P=2$ when $R=5$ and $Q=4$, find:

(i) The law connecting P, Q and R. *(3 marks)*

(ii) The positive value of Q when $P=4.5$ and $R=5$. *(2 marks)*

(c) Make Q the subject in the law connecting P, Q and R in (b) (i) above. *(2 marks)*

6. P varies directly as the square of R and inversely as the square root of Q.

(a) Find the % change in P if R is increased in the ratio $3:2$ and Q is decreased by 19%. *(4 marks)*

(b) Given that $P=7.2$ when $R=2.4$ and $Q=1.44$,

(i) Find the equation connecting P, Q and R. *(3 marks)*

(ii) Find the value of Q when $P=6$ and $R=1.4$. *(3 marks)*

7. Three quantities m,n and p are such that m is directly proportional to the cube of root of n and n varies inversely as the square of p.

(a) If $m=1$ when $n=8$ and $p=0.6$, find:

(i) the relationship between m and n. *(2 marks)*

(ii) the relationship between n and p. *(2 marks)*

(iii) the value of m when $p=\dfrac{2\sqrt{6}}{15}$. *(3 marks)*

(b) If n is multiplied by 0.125, find the percentage change in p. *(3 marks)*

8. The mass, m grams of a cylinder varies jointly as the square of its radius, r, and its height, h.

(a) Find the % change in the mass if the radius is trebled and the height is halved. *(3 marks)*

(b) Given that $m=990g$ when $r=3cm$ and $h=7cm$;

(i) Find the equation connecting m, r and h *(3 marks)*

(ii) Calculate the value of m when $r=3.5cm$ and $h=5cm$. *(2 marks)*

(c) Taking $\pi=\dfrac{22}{7}$, calculate the density of the cylinder. *(2 marks)*

9. A quantity A varies as the square of B. B on the other hand, is partly constant and partly varies as the square root of C.

(a) Using three constants k,m and n, write an expression that relates A with C. *(3 marks)*

(b) Given that $A=10$ when $C=4$, $A=23$ when $C=9$ and $A=42$ when $C=16$, find the equation connecting A and C. *(4 marks)*

(c) Find the value of C when $A=15.75$. *(3 marks)*

10. Two variables P and Q are such that P varies partly as the square of Q and partly as the inverse of Q. $P=10.5$ when $Q=2.5$ and $P=30.75$ when $Q=4$.

(a) Find the equation connecting P and Q. *(4 marks)*

(b) Find P when $Q=1.5$ *(2 marks)*

(c) Q is directly proportional to the square root of R, and $Q=3.04$ when $R=3.61$, find:

(i) The relationship between Q and R. *(2 marks)*

(ii) The relationship between P and R. *(2 marks)*

Answers

Formulae

1. $n=\dfrac{ax^2-m^3w}{1-m^3}$

2. $M=\pm\sqrt{\dfrac{NX^2}{N^2-X^2}}$

3. $x=-\dfrac{3r}{4}$

4. $R=\sqrt{\dfrac{A+\pi r^2}{\pi}}$

5. $t=\dfrac{xL^2\sqrt{L}}{L^2+x^2\sqrt{L}}$

6. $x=\dfrac{\pm\sqrt{a^2m^2+b^2m}}{b}$

7. $h=\dfrac{E^2+0.5x^2}{E^2+x^2}$

8. $R=\sqrt[3]{\dfrac{m^3C}{m^3-C^3}}$

9. $b=\pm\sqrt{\dfrac{a-t^2}{at^2+1}}$

10. $\dfrac{1}{x}=\dfrac{mt^3+b}{nt^3+ay}$

11. $k=\dfrac{1+2y}{t^2}$

12. $c=\dfrac{A^2b}{\pm\sqrt{A^4-1}}$

13. $x=\dfrac{2w-12p^2r}{16p^2-1}$

14. $x=\dfrac{\pm\sqrt{9L^2y+4PT}}{2}$

15. $Q=\dfrac{T}{\pm\sqrt{T^2-P^2}}$

16. $p=\log_A\left(\dfrac{d^3q-q}{d^3}\right)$

17. $P=\dfrac{T}{\pm\sqrt{K-T^2K}}$

18. $p=\dfrac{qr}{q+r}$

19. $p=\log_r(1+3q)-1$

20. $a^x\left(a^2-1\right)=a(a-1)$ (hint)

Variation (Section I)

1. Decreased by 29.6%

2. $s=2t+4\sqrt{t}, s=30$

3. $131\frac{13}{27}\%$

4. $m=20+5t^2, m=265$

5. $P=\dfrac{12Q^2}{\sqrt{R}}$

6. $A=2B+4\sqrt{B}, A=12.58$

7. $Q=600-10E^2, E=2.5$

8. $P=50-\dfrac{3}{Q^2}, P=49.88$

9. $y=\dfrac{900}{x^2}$

10. 23.52

11. Decreased by $46\frac{2}{3}\%$

12. $M=100N+\frac{1}{2}N^2$

13. $mn=12$

14. 58.4%

15. 1575

16. 32

17. 37.8125%

18. Decreased by $87\frac{1}{2}\%$

19. 0.8

20. $t=\dfrac{20}{s\sqrt{s}}$

Variation (Section II)

1. (a) $R=\dfrac{16S}{\sqrt{T}}$ (b) (i) $R=3840$ (ii) 14.13%

2. (a) $C=60n+\dfrac{300}{n}$ (b) 630 (c) 15

3. (a) $A=2B+4\sqrt{B}$ (b) (i) 96 (ii) 4.41

4. (a) 25000, 400 (b) $C=25000+400n$ (c) (i) 35000 (ii) 37 (d) 30

5. (a) 60% (b) (i) $P=\dfrac{0.625Q^2}{R}$ (ii) 6 (c) $Q=4\sqrt{\dfrac{PS}{10}}$

6. (a) 150 (b) (i) $P=\dfrac{1.5R^2}{\sqrt{Q}}$ (ii) $Q=0.2401$

7. (a) (i) $m=\dfrac{1}{2}\sqrt[3]{n}$ (ii) $n=\dfrac{2.88}{P^2}$ (iii) 1.5 (b) 182.84%

8. (a) 350% (b) (i) $m=15\frac{5}{7}r^2h$ (ii) $m=962.5g$ (c) $\rho=5$ g/cm^3

9. (a) $A=k+m\sqrt{C}+nC$ (b) $A=2-2\sqrt{C}+3C$ (c) 6.25

10. (a) $P=2Q^2-\dfrac{5}{Q}$ (b) $P=1\dfrac{1}{6}$ (c) $P=5.12R-\dfrac{3.125}{\sqrt{R}}$